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THESIS

ANALYSIS OF DECISIONS MADE USING THE ANALYTIC HIERARCHY PROCESS

by

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September 2013

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ANALYSIS OF DECISIONS MADE USING THE ANALYTIC HIERARCHY PROCESS

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Lists of Acronyms and Abbreviations

ARMS Advanced Robotic Manipulator System

AHP Analytic Hierarchy Process

COA Courses of Action

FMHR Field Material Handling Robot

FGDO Foreign Government Defense Organizations

MCDM Multi Criteria Decision Making

OR Operations Research

RDTE Research, Development, Test, and Evaluation

RTCH Rough Terrain Cargo Handlers

DoD United States Department of Defense

USDCH Universal Self-Deployable Cargo Handler

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Executive Summary

The Analytic Hierarchy Process (AHP) is a tool used by decision-makers, including many military ones. The output of the AHP lends itself to analysis to gain much information about the decision-maker's priorities and thought processes.

The information that can be directly gained from analysis of the output of the AHP include the factors a particular decision-maker relies on to make decisions, and how those factors compare to each other in priority. In addition, the decision itself can be examined for susceptibility to rank reversal. Knowing whether rank reversal is possible in a decision aids in being able to predict the outcome of a decision with new inputs.

Over and above the information that can be directly gained from the output of the AHP, two techniques can be employed, based on the knowledge gained from analyzing the output, in order to exploit AHP decisions. These techniques are overestimation and underestimation.

Underestimation allows the determination of the optimal specifications of a new alternative that will cause a rival decision-maker to detrimentally forgo the commitment of additional resources in response to an altered decision.

Overestimation allows the determination of the optimal specifications of a new alternative that will cause a rival decision-maker to needlessly commit resources in response to the altered decision. This can be done while minimizing the required resources needed to produce this new alternative.

Two case studies, one unclassified and one classified, demonstrate the viability the use of these techniques in operational and strategic decision making scenarios.

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CHAPTER 1:
Introduction

This thesis analyzes the use of the Analytic Hierarchy Process (AHP) as a decision-making tool. The analysis shows what information can be gained about a military decision-maker who uses the AHP, and how this information can be utilized, permitting U.S. and allied forces to execute efficient and effective military operations. A case study of a decision made by a military decision-maker through the use of the AHP demonstrates techniques that can be used to garner information about the decision-maker.

1.1 The Problem of Mirror Imaging

In Dien Bien Phu, 1954, the colonial French army, attempting to cut the Vietminh supply lines into Laos, chose artillery sites considered heavily defensible. The sites were surrounded by hills covered in thick jungle. These hills, the French surmised, would be inaccessible to Vietminh artillery, as there was no logistics network in place to move heavy artillery guns to the hilltops. What they did not consider was the Vietminh's ability to break the guns down and transport them along jungle paths by hand (Windrow, 1998). This ultimately led to the Vietminh outmatching the French in artillery numbers, leading to the French defeat at Dien Bien Phu. This was a historic defeat, since a small, poorly equipped and ill-trained military was able to defeat a first world power. France's failure at Dien Bien Phu was primarily a result of the cognitive trap of mirror imaging: assuming that the Vietminh would make the same logistics decisions as the French.

1.1.1 What is Mirror Imaging?

Mirror imaging is a form of decision-making in which one party assumes another will act in the same way they would when making a decision. This is commonly seen in military operations when a commander assesses his/her enemy's options to be the same as their own, and then assumes the enemy will act in the same manner as he/she would. In the example above, the French assessed the surrounding hills to be too rugged for transporting artillery to the top, so they surmised that the Vietminh would be similarly limited. The outcome of the battle illustrates one of the dangers of mirror imaging in a combat situation.

It should be noted that mirror imaging need not occur only between aggressors. Mirror imaging can have negative impacts on cooperative engagements as well. If partner A assumes that partner B will act in a certain way, and plans accordingly, the mutual plan can suffer when B does something different. Mirror imaging stems from a lack of knowledge about the other party's capabilities or thought processes; the resolution of this effect is to gain a deeper understanding of both. In the cooperation scenario, this is easily achieved by both parties sharing all pertinent information on capabilities and thought processes; which is mutually beneficial and avoids the pitfalls of mirror imaging.

1.1.2 Mirror Imaging Loses Battles

In a competitive situation, mirror imaging is a potentially dangerous situation. A commander may make choices based on erroneous assumptions about their enemy's actions. In our example, the French commander decided, "The Vietminh could not get artillery onto those hills, so we do not need to increase our numbers to defend against them." However, the enemy may have different tactics available to them, ones that the commander has never thought of, which will ultimately lead to the commander's defeat. If the commander had a more complete understanding of the enemy's capabilities and thought processes, he would not have relied on mirror imaging, and his actions would have been different. The French commander may have realized, "I assume the Vietminh can get artillery on those hills, so I will increase my numbers and defeat them if they do." Now, using a strategy based on avoiding mirror imaging, the French commander can achieve victory. This is a very generalized and simple illustration, but the principle is this: The more information one has about an opponent's capabilities and thought processes, the more accurately one can predict their actions. This knowledge can lead to a profitable outcome, including military victories in hostile situations.

1.1.3 Mirror Imaging Based Mistakes are Costly and Hard-learned

The common means of learning of the downfall of mirror imaging is to fall victim to it, and reap the bloody benefit of the information gained. This is a costly method, as it means losing battles, personnel, and equipment, for the sake of gaining insight. There is no single solution to mirror imaging; every enemy will have different information or thought processes, and both parties have a vested interest in not letting the other side gain understanding of their capabilities and thought processes. This combination makes mirror imaging an extremely difficult, widespread problem.

This thesis will examine a case study example to offer techniques to combat mirror imaging. This case study will show how analyzing the output of an AHP based decision is a good method to aid in combating mirror imaging. The AHP lends itself well to analysis, as it is transparent, simple to compute, and easily manipulated. The AHP is a decision-making process that divides a decision into its critical elements, referred to as factors. These factors are then compared to each other by a group of experts to assign a weight to each factor. Finally, each alternative being considered for the decision is then measured according to the factors, and a score is calculated. These scores determine which alternative is considered best. The AHP method clearly delineates what the decision-maker's thought process and priorities are. Understanding a decision another party has made using the AHP will help commanders make informed decisions about future actions which, in turn, minimizes loss to personnel and equipment, and maximizes battle success.

1.1.4 The History of the Analytic Hierarchy Process

In the late 1960s, Dr. Thomas Saaty, one of the pioneers of Operations Research (OR), was directing research projects for the Arms Control and Disarmament Agency at the U.S. Department of State. These projects were designed to implement arms control strategies and policies. They brought together many of the country's top scientists, to formulate ways to reduce arms numbers, and lawyers, to interpret the laws governing arms control. In spite of the talents of the people Dr. Saaty recruited, he was disappointed in the results of the team's efforts (Foreman & Gass, 2001). Dr. Saaty later recalled the following:

> Two things stand out in my mind from that experience. The first is that the theories and models of the scientists were often too general and abstract to be adaptable to particular weapon trade off needs. It was difficult for those who prepared the U.S. position to include their diverse concerns. . . and to come up with practical and sharp answers. The second is that the U.S. position was prepared by lawyers who had a great understanding of legal matters, but [who] were not better than the scientists in assessing the value of the weapon systems to be traded off (Foreman & Gass 1996, p. 5).

In subsequent years, Dr. Saaty developed a tool not just for the scientists and lawyers to use, but useful in a wide range of applications by a variety of professionals. The tool developed, the AHP, is a method of Multi Criteria Decision Making (MCDM). The AHP uses the decision-maker's opinions to develop a priority list for the factors that make up a decision, as well as how each alternative option weighs against those factors.

The AHP is widely used throughout the world in various fields of study for MCDM. A few examples are: deciding how best to reduce the impact of global climate change (**?**) (Berrittella, Certa, Enea, & Zito 2007), quantifying the overall quality of software systems (Mccaffrey, 2005), selecting university faculty (Grandzol, 2005), deciding where to locate offshore manufacturing plants (Atthirawong MacCarthy, & Gregory, 2002), assessing risk in operating cross-country petroleum pipelines (Dey, 2003), deciding how best to manage U.S. watersheds (De Steiguer, Duberstein, and Lopes, 2003), and the U. S. Army choosing the best military rough terrain cargo handler (Bard & Sousk, 1990).

1.1.5 AHP in the United States

Aside from a few instances in the U.S. Army, the United States Department of Defense (DoD) does not use the AHP often. This is not due to any perceived flaw in the AHP, but because of a difference in decision-making fundamentals. The DoD prefers the use of methods that leverage its experience in operational matters, using experience as a guide in predicting operations. This leads to the use of simulation, both computer-aided and not, or Courses of Action (COA) development, which also involves the use of simulation. Simulation has its own set of pros and cons. Among the pros are the ability to consider many options in a single simulation, and to change the input parameters easily in order to view many different situations at once. The con is the pitfall of believing the output of the simulation to be truth and not possibilities. This misplaced belief can still lead to mirror imaging, which we have shown to be a dangerous decision-making method.

Although the DoD does not currently use the AHP often in decision-making, many Foreign Government Defense Organizations (FGDO) do. This difference in decision-making processes between the DoD and FGDO is one which can aid the DoD in gaining an understanding of them. By being aware of FGDO's processes, the DoD can improve future cooperative or competitive engagements. Therefore, a deep understanding of the AHP and of the decisions that are made using this process is essential in understanding the capabilities and thought processes of many FGDOs. This understanding will aid in both cooperating with our allies and in competing with our adversaries.

1.2 Understanding Decision-making Processes

This thesis does not propose to solve the entire issue of mirror imaging, or to give a cut-and-dried tool to use in analyzing another party's decisions that applies to every situation. There are, however, several options one can use to counteract some of the issues of mirror imaging in military scenarios. These include simulating the situation to flesh out all alternatives available to the enemy, producing COAs that cover all possibilities; or analyzing an enemy's prior decisions in an attempt to predict their actions. The latter option is done most easily with a decision made using the AHP, as it lends itself so well to analysis. Decisions made using the AHP are ideal for analysis because the AHP has a very prescribed methodology, and most of the information that goes into the decision, i.e., the decision-maker's though process, is presented in the output. It can be difficult to determine these critical pieces of a decision with a simulation or COA development reference, as the inputs to these two methods are not always transparent.

To address the issue of mirror imaging, and present a set of solution techniques, this thesis examines a two case studies of the output of an AHP decision made by military organizations, one unclassified, one classified. This analysis will show what capabilities the military organization has, the alternatives being considered for the decision, and the thought process used to determine what parts of the decision are more or less important. There are two possible outcomes to this type of analysis: either new information is obtained that alters what was known, or old information is confirmed, solidifying preconceived notions. Either way, better understanding is achieved, and mirror imaging is less likely. This information can be used to enhance military efforts by allowing military commanders to better predict their opponents actions, leading to victories.

This thesis will examine a decision made using the AHP in two parts. The first part will be analysis of the decision, the decision hierarchy, the alternative's hierarchy, and the results, in order to extract what capabilities and thought processes can be gleaned from the decision. The second part of the examination will be an analysis of the ways the decision can be exploited. This analysis will consist of checking the decision output's resistance to rank reversal, developing a method for inserting an alternative into the output list at a specific point, and using the decision's mechanic to pick what information about a new alternative will be shared with the decision-maker.

1.3 Thesis Outline

The following chapters are broken down thus: Chapter II covers an in depth history and mechanics of the AHP, as well as a literature review of its strengths and weaknesses; Chapter III details the selected example problem and the method used for analyzing said problem; Chapter IV presents the results of the analysis; and Chapter V provides conclusions and recommendations.

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CHAPTER 2:
The Analytic Hierarchy Process

The previous chapter established that mirror imaging is a potentially fatal mental trap that decision-makers, especially military decision-makers, can fall into. One defense against this trap is gathering information on an opposing decision-maker's thought process and capabilities. One way to gather this information is through analyzing the output of the AHP, a process which lends itself well to gathering information on what the user's priorities and thought processes are. This chapter describes the AHP in detail.

2.1 The Process

In his book, *Decision Making for Leaders: The Analytic Hierarchy Process for Decisions in a Complex World*, Dr. Thomas Saaty breaks the AHP into five steps, as follows (Saaty, 2001):

- Step 1 - Build a hierarchy
- Step 2 - Make comparisons
- Step 3 - Calculate weights
- Step 4 - Check consistency
- Step 5 - Produce result

Before beginning the in-depth explanation of the AHP, a few terms frequently used in the process need to be defined.

- *Hierarchy* - The representation of a decision broken down into its constituent pieces, here called factors.
- *Factors* - The constituent pieces that combine to make a decision. These are broad categories, often lacking detail.
- *Sub-factors* - The pieces of factors broken down to smaller pieces. As factors are to the decision, sub-factors are to factors. Depending how in depth the decision-maker wants to go, sub-factors can be divided into sub-sub-factors, and etc.
- *Weights* - The numerical interpretation of how important a factor is to the decision, usually reported as a decimal. All of the factors in a decision will sum up to 1. Sub-factors can have weights as well, representing how important that sub-factor is to its factor.

- *Consistency* - A measure of how close a matrix is to obeying the rule of consistency, that if A is twice as important as B, and B is twice as important as C, then A must be four times as important as C.
- n - the number of factors or sub-factors used in the final composition of the decision hierarchy.

The best way to describe the AHP is to use an example. For illustration purposes, consider a basic decision of which ship is superior, Ship I, Ship II or Ship III, with the parameters found in Table 2.1:

Table 2.1: Constructed parameters for the three ships used in the example decision: Which is the best ship?

	Purchase Cost	Sustainment Cost	Cruising Speed	Top Speed
Ship I	$10M	$400k	15	25
Ship II	$20M	$1,000k	15	20
Ship III	$30M	$2,000k	20	30

A graphical representation of the decision broken down into factors and sub-factors, along with each alternative, is found in Figure 2.1.

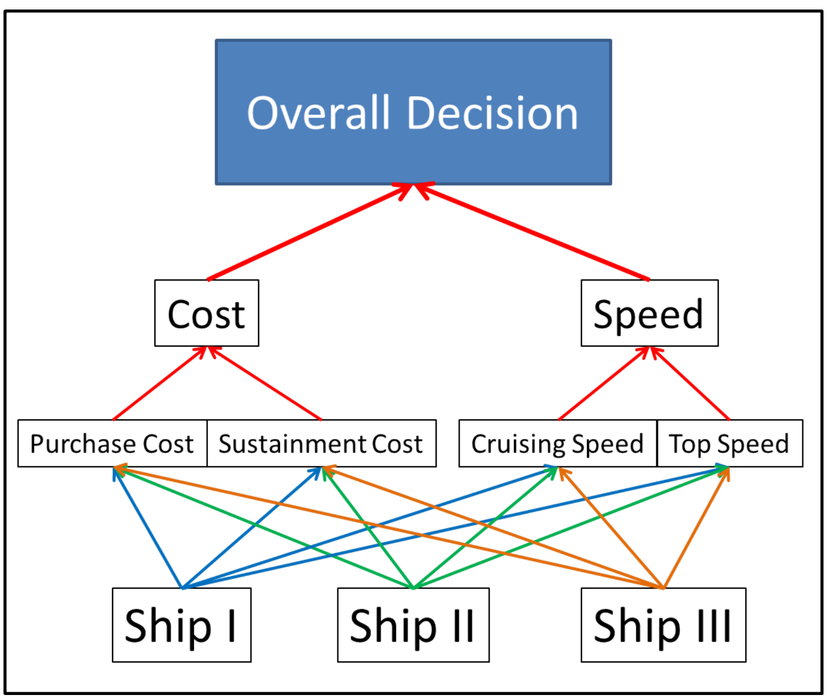

Figure 2.1: Full decision breakdown showing each alternative and its relation to the decision's factors.

2.1.1 Build a Hierarchy

To begin the AHP process, first determine the parts of the decision that contribute to the overall outcome. To do this, divide the decision into broad categories, then subdivide them into smaller pieces. For instance, the ship comparison example decision could be divided into the factors *cost* and *speed*. The *cost* factor could be divided into sub-factors *purchase cost* and *sustainment cost* and *speed* could be divided into sub-factors *cruising speed* and *top speed*. See Figure 2.1.1 for a graphical depiction of this. For this example, there would be $n = 4$ sub-factors to compare.

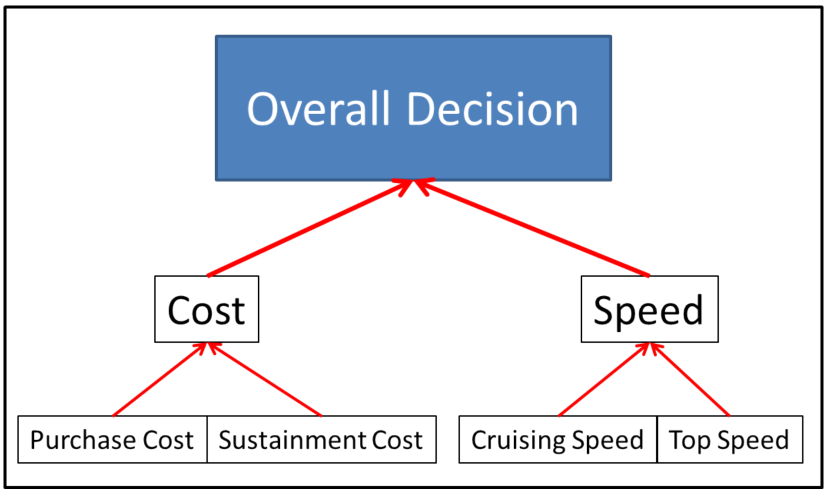

Figure 2.2: Representation of the basic example decision, showing the breakdown of the larger decision into its factors and sub-factors.

2.1.2 Make Comparisons

Once the factors and sub-factors have been determined, pairwise comparisons of the factors will need to be made to determine how much each factor contributes to the decision. These comparisons will be made by a panel of subject matter experts chosen by the decision-maker for this purpose. The number and composition of the expert panel is not defined. There can be any number of experts, balancing the experience gained from a large panel, with the ability to reach consensus of small panel. Likewise, diversity in the backgrounds of the panel will increase the knowledge brought to the decision, but could make consensus harder to achieve.

There are two ways that the comparisons can be made. The first way is for each expert to assert his/her opinion, and then some method (averaging, weighting, determining means) is used to achieve a single value to represent the group. The second method is for the panel to discuss the decision, and achieve a consensus number together. Whether every expert compiles their own matrix of comparisons, or all experts decide on a general consensus, the necessary result is a single matrix of pairwise comparisons. In our example this would be a 4 x 4 matrix like the one found in Table 2.7.

In order to make the pairwise comparisons, Dr. Saaty developed a 9-point scale. The scale is based on determining whether one factor compared to another is *equally important* (1), *slightly more important* (3), *more important* (5), *greatly more important* (7), or *absolutely more important* (9). Values of 2, 4, 6, and 8 are reserved for intermediate values. The reciprocal value is achieved when comparing the factors in the opposite direction, i.e., if A is *more important* than B (5), B could be said to be *less important* than A ($\frac{1}{5}$).

Ultimately, each of the four sub-factors above will need to be compared to each other. There are two ways this can be done. In the first method, the factors of *cost* and *speed* are compared to each other, followed by comparing each sub-factor under each factor separately. Following the example, define *cost* as *slightly more important* (3 on the 9 point scale) than *speed*. The two by two matrix this would produce is shown in Table 2.2. Looking at the *cost* factor, let us define *purchase cost* as between *slightly* and *more important* than *sustainment cost* (4 on the 9 point scale). For the speed factor, define *cruising speed* and *top speed* as *equally important* (1 on the 9 point scale).

Table 2.2: Matrix of the comparisons of the factors in the overall decision.

OVERALL	Cost	Speed
Cost	1	3
Speed	$\frac{1}{3}$	1

The other option to determine each sub-factor's overall contribution to the decision is to directly compare each sub-factor to every other. This method involves more comparisons, and can become confusing, as a comparison could be made between two very dissimilar sub-factors, leading to an uncertain result. For instance, trying to compare *cruising speed* to *sustainment cost* would be difficult, as the two sub-factors are very dissimilar. This second method can also lead to increased inconsistency, as will be discussed below.

2.1.3 Calculate Weights

The calculation of the weight of each factor or sub-factor is an exercise in matrix mathematics. The vector of weights is the normalized eigenvector of the matrix associated with the largest eigenvalue, λ_{Max}, of the matrix. The weights for the pairwise comparison matrix in Table 2.2 are given in Table 2.3.

Table 2.3: Matrix of the comparisons of the factors in the overall decision, with the weight of each factor included.

OVERALL	Cost	Speed	**Weight**
Cost	1	3	**0.75**
Speed	$\frac{1}{3}$	1	**0.25**

Tables 2.4 and 2.5 represent the comparisons for *cost* and *speed* respectively. See Figure 2.1.3 for a graphical depiction of all of weights.

Table 2.4: Matrix of the comparisons of the sub-factors in the *cost* factor, with the weight of each sub-factor included.

COST	Purchase	Sustainment	**Weight**
Purchase	1	4	**0.8**
Sustainment	$\frac{1}{4}$	1	**0.2**

Table 2.5: Matrix of the comparisons of the sub-factors in the *speed* factor, with the weight of each sub-factor included.

SPEED	Cruising	Top	**Weight**
Cruising	1	1	**0.5**
Top	1	1	**0.5**

The next step in determining each sub-factor's contribution to the overall decision is simply to multiply each sub-factor's value by its corresponding factor's value. To illustrate, consider the *purchase cost* sub-factor, where:

- $OverallValue_{PC}$ = The contribution of purchase cost to the overall decision.
- $Value_{PC}$ = The contribution of purchase cost to the cost factor.
- $Value_{C}$ = The contribution of cost to the overall decision.

$$OverallValue_{PC} = Value_{PC} * Value_{C} = 0.8 * 0.75 = 0.6 \tag{2.1}$$

The rest of the sub-factors' overall contributions are tabulated in Table 2.6. In addition, Table 2.6 shows the matrix of pairwise comparison values that result from the calculated weights. These values were calculated by using Equation 2.2. The weights are also graphically depicted in

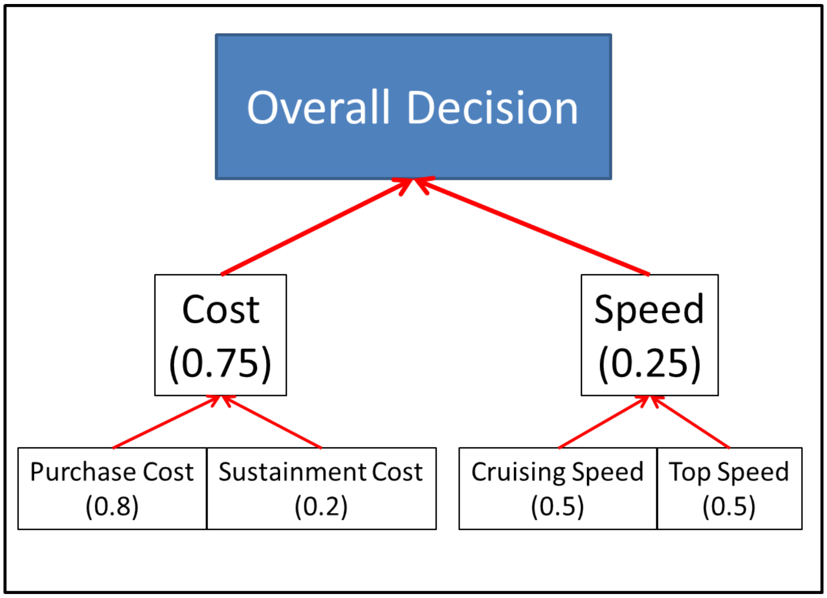

Figure 2.3: Representation of the example decision with each factor broken into its sub-factors. Factor values have been included in parenthesis.

Figure 2.1.3. The calculated weights tell the decision-maker how much each factor contributes to the overall decision. For instance, in our example, *price* makes up 60 percent of the decision of which ship is best.

$$ComparisonValue = \frac{Subfactor_{Column}weight}{Subfactor_{Row}weight} \tag{2.2}$$

For the comparison of the factors *Purchase Cost* and *Cruising Speed* this would be $\frac{0.6}{0.125} = 4.8$.

Table 2.6: Pairwise comparisons of sub-factors with associated weights.

	Purchase Cost	Sustainment Cost	Cruising Speed	Top Speed	**Factor**
Purchase Cost	1	4	4.8	4.8	**0.6**
Sustainment	0.25	1	1.2	1.2	**0.15**
Cruising	0.21	0.83	1	1	**0.125**
Top	0.21	0.83	1	1	**0.125**

Returning to the issue mentioned above on the difficulty in comparing each sub-factor to every other, consider this: had that method been chosen to do the comparisons here, an issue would arise, in that not all of the comparisons are whole numbers. To illustrate this, Table 2.6 has been reproduced in Table 2.7, with each comparison value being the rounded value from Table 2.6 to conform to Dr. Saaty's 9 point scale. The drawback involved with this method will be described

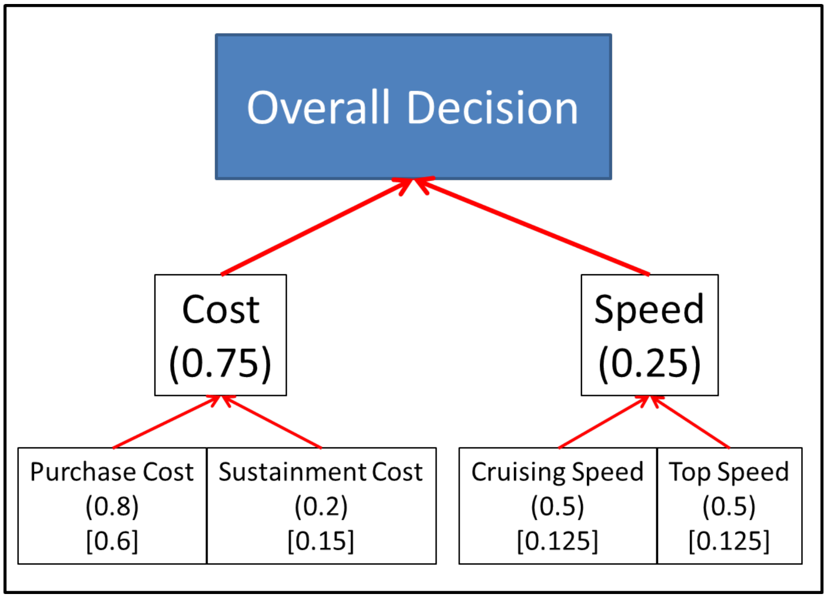

Figure 2.4: Representation of the example decision broken down to minor sub-factors with each sub-factor's contribution to its factor in parenthesis and its overall weight in square brackets.

in the next section.

Table 2.7: Pairwise comparisons using rounded numbers to conform to Dr. Saaty's 9 point index.

	Purchase Cost	Sustainment Cost	Cruising Speed	Top Speed	**Factor**
Purchase Cost	1	4	5	5	**0.612**
Sustainment	0.25	1	1	1	**0.137**
Cruising	0.2	1	1	1	**0.13**
Top	0.2	1	1	1	**0.13**

2.1.4 Check Consistency

Once all of the overall weights have been calculated, the next step of the AHP is to check the factor matrix for consistency. Consistency in the matrix is important, because it reflects how precise the result of the process will be. The basic idea of consistency is that for any given n by n matrix:

	Factor 1	$\cdots$	Factor N
Factor 1	a_{11}	...	a_{1n}
$\vdots$	$\vdots$	$\ddots$	$\vdots$
Factor N	a_{n1}	$\cdots$	a_{nn}

and for any $i, j, k \leq n$, the equation $a_{ij} * a_{jk} = a_{ik}$ must hold true. Put into plain language, if factor X is two times more important than factor Y, and factor Y is two times more important than factor Z, factor X must be four times more important than factor Z. Inspecting all of the entries in Table 2.6 shows complete consistency. To illustrate the consistency calculations, the matrix in Table 2.7 will now be used.

To calculate inconsistency, Dr. Saaty suggests using the following procedure: First, using the previously determined maximum eigenvalue of the matrix, λ_{max}, calculate the consistency index, CI, based on Equation 2.3, where n is the number of factors used in the decision (Saaty, 2001):

$$CI = \frac{\lambda_{max} - n}{n - 1} \tag{2.3}$$

For the matrix in Table 2.7, $\lambda_{max} = 4.00623$, and $n = 4$, resulting in $CI = 0.0021$. Next, using the calculated CI, determine the matrix's consistency ratio, CR from the equation:

$$CR = \frac{CI}{RI} \tag{2.4}$$

Here, RI is the random consistency index created by Dr. Saaty and used in all consistency calculations. This index is based on the idea that a randomly generated n x n factor comparison matrix, based on the 9 point scale, will generate a certain amount of inconsistency. For values of n up to 10, Dr. Saaty randomly generated 500 matrices and calculated their average inconsistency, to use as a comparison index that he called RI (Saaty, 2001). The table that Dr. Saaty produced is seen in Table 2.8.

Table 2.8: Random Consistency Index as reported by Dr. Saaty (After Saaty 2001).

n	1	2	3	4	5	6	7	8	9	10
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

If the value of CR is less than 10 percent, i.e. the factor matrix in use has less than 10 percent of the inconsistency found in a random factor matrix, then the matrix is consistent enough and can be used for calculating results. If, however, the CR is higher than 10 percent, the pairwise comparisons should be redone by the expert panel in order to increase the consistency of the final decision. Using Table 2.7's *consistency index* of 0.021 percent, and $n = 4$, it can be seen that the sample problem's *random consistency index* is 0.023 percent, much less than 10 percent. This means that the final decision from this process will produce a result consistent with the thoughts of the expert panel.

2.1.5 Produce Result

Now that the factors' weights have been calculated and consistency has been determined to be within acceptable limits, the final step in the AHP is to determine each alternative's value for each factor. For instance, in the factor *purchase cost* we have the values of \$10M (Ship I), \$20M (Ship II), and \$30M (Ship III). The question is: how important is a smaller purchase cost? This is a question that can be answered in several ways. The first way is to submit the alternative's parameters to the same pairwise comparison process used for the factor's weights, where elements of the same factor are compared, say *purchase cost*. This method may be preferred if there are many constraints on the factor, for instance, a \$20M budget, or the requirement to buy as many ships as possible with \$35M. In order to make the comparisons more logical, the verbiage of the comparison could be changed to *acceptable* vice *important*, i.e., a \$10M purchase cost is *more acceptable* than a \$20M purchase cost (5). The result of this would be a list of weights for each alternative's contribution in each of the factor areas that adds up to 1. Table 2.9 below shows what the comparison matrix would look like for *purchase cost*, where:

- \$10M is between *equally as acceptable* as and *slightly more acceptable* than \$20M
- \$10M is between *slightly more acceptable* and *more acceptable* than \$30M
- \$20M is between *equally as acceptable* as and *slightly more acceptable* than \$10M

Table 2.9: Example pairwise comparison of alternative's attribute under a single factor, here *purchase cost*.

PURCHASE COST	Ship I (\$10M)	Ship II (\$20M)	Ship III (\$30M)	Result
Ship I (\$10M)	1	2	4	0.57
Ship II (\$20M)	$\frac{1}{2}$	1	2	0.29
Ship III (\$30M)	$\frac{1}{4}$	$\frac{1}{1.5}$	1	0.14

A second method for obtaining the values is to use a utility function. A utility function allows the decision-maker to set boundaries on the parameters, or to use a scale that is other than linear. This utility function method has a few advantages over the first. One, no more subjective comparisons will have to be made, which can save time and effort on the part of the expert panel. Two, the functions can be reused when new alternatives are submitted, again saving time. For our example problem, a simple normalized linear utility function has been used to convert the ship parameter matrix in Table 2.1 to the utility matrix in Table 2.10.

Table 2.10: Relative values for the factors of each alternative in the sample decision, using simple linear utility functions.

	COST		**SPEED**	
	Purchase Cost	Sustainment	Cruising	Top
Ship I	0.67	0.62	0.0	0.33
Ship II	0.33	0.38	0.0	0.0
Ship III	0.0	0.0	1.0	0.67

Once these relative values have been calculated for the alternatives, all that remains is to calculate each alternative's value to the overall decision. This is done by using Equation (2.5) where:

- $Value$ = The alternative's value in the overall decision.
- $Factor_i$ = Factor i's contribution to the overall decision.
- $AlternativeValue_i$ = The alternative's relative value for factor i.
- k = The number of factors in the decision.

$$Value = \sum_{i=1}^{k} (Factor_i * AlternativeValue_i) \tag{2.5}$$

There are two possibilities when comparing alternatives using the AHP. First, that alternative A's value is equal to alternative B's. By the theory of the AHP, this would mean that it makes no difference which alternative is chosen between the two, as they will each satisfy the decision with the same weight. The other possibility is that alternative A's value is different than alternative B's. The higher value will always determine which alternative is superior. The final values for each alternative in the sample problem are listed in Table 2.11, showing that Ship I is the superior alternative. Note, for this calculation, the weights from Table 2.6, as they are the most consistent.

Table 2.11: Parameters for the ships used in the example problem.

	FACTOR	Ship I	Ship II	Ship III
Purchase Cost	**.60**	0.67	0.33	0.0
Sustainment	**0.15**	0.62	0.38	0.0
Cruising	**0.13**	0	0	1.0
Top	**0.13**	0.33	0	0.67
VALUE		**0.53**	**0.26**	**0.21**

2.2 Literature Review: The Benefits of the AHP

The AHP has many benefits to its use. Primarily these fall under the heading of managing chaos. Specifically, the AHP is a tool that can be used to simplify and synthesize complexity. In addition, the AHP is applicable to a wide range of decisions.

2.2.1 Simplifying Complexity

Dr. Saaty sought, in creating the AHP, a simple way to deal with complexity. Simple enough so that lay people with no formal training could understand and participate. He found one thing common in numerous examples of the ways humans had dealt with complexity over the ages – that was the hierarchical structuring of complexity into homogeneous clusters of factors (Foreman & Gass, 2001).

2.2.2 Synthesis of Complexity

In addition to being able to break a decision down into its constituent factors, the AHP offers an additional benefit, the ability to combine different expert's analysis. A decision-maker may have multiple different experts working for them, executives in a corporation, commanders in a military, etc., and each expert may have their own analysis of the decision being considered. The AHP allows for diversity in these analyses and makes it possible to combine them into a unified decision, based on the mathematics involved in calculating the weights of factors (Foreman & Gass, 2001, pg. 470).

2.2.3 Applicability

The AHP, due to its comprehensive nature and ability to turn big decisions into a series of small determinations, has a wide range of applications. The AHP can be used for resource allocation, choosing among alternatives, and process engineering, to name a few (Foreman & Gass, 2001, pg. 471).

2.3 Literature Review: Critiques of the AHP

Contrasting the many benefits of the AHP are the flaws that some have found in it. The major flaws that have been noted in numerous articles written since the AHP was first published are its use of the linear 1 to 9 scale, and the issue of rank reversal. These two issues are mildly related, as the inconsistency generated by the linear scale can promote rank reversal.

2.3.1 Linear and Rigid Scale

The first issue commonly reported in literature on the AHP has to do with Dr. Saaty's choice of a linear 1 to 9 scale for use in the pairwise comparisons. The main issue descends from the possible inconsistency generated from comparisons. For instance, if factor A is determined to be *slightly more important* than B (3), and B is determined to be *slightly more important* than C (3), then to assure complete consistency, A would need to be rated as *absolutely more important* than C ($3x3 = 9$). This is a big jump for two factors that are only slightly more important than each other. Following logically, if any two comparisons, A to B and B to C, are more than *slightly more important*, the resultant third comparison, A to C, will induce inconsistency, since 9 is the highest rating possible for any comparison. In *Some Comments on the Analytic Hierarchy Process*, R.D. Holder proposes the use of an exponential scale to resolve the issue (Holder, 1990, pg. 1073-1074). Essentially, one needs to determine the minimum discernible difference in factors, α. Then, comparisons become exponential factors of α:

- α^0: *equally important*
- α^2: *slightly more important*
- α^4: *more important*
- α^6: *greatly more important*
- α^8: *absolutely more important*

Now, if factor A is determined to be *slightly more important* than B (α^2), and B is determined to be *slightly more important* than C (α^2), A will be rated as *more important* than C ($\alpha^2 * \alpha^2 = \alpha^4$), which makes more sense according to the verbiage descriptions.

Dr. Saaty responded to Mr. Holder's critiques in the 1991 article "*Response to Holder's Comments on the Analytic Hierarchy Process*" in the Journal of the Operations Research Society. Dr. Saaty states, "He [Holder] forgets that the AHP is a theory for the human level of coping and not a number-crunching device for measuring a single attribute from zero to infinity" (Saaty, 1991, pg. 911). Dr. Saaty's point here is that the important thing is that the expert panel be easily able to convey their priorities to the verbiage of the scale, not necessarily that the scale meet a rigorous mathematical definition.

2.3.2 Rank Reversal

The second, and possibly more serious, issue is that the application of the AHP can result in rank reversal. Rank reversal occurs when a new alternative is added to the list of alternatives, or one of the existing ones is removed, and when the process is re-run, a new order is seen among the alternatives that were included originally. The commonly used scenario is this: the waiter asks if you want chicken or fish, and you reply fish. The waiter then remembers that steak is also available, and you now want chicken. The addition of the third alternative, which does not make it through the decision process as the best option, should not have altered the order of the two existing options.

To illustrate rank reversal, consider this change to the above example: To simplify, the only two criteria for choosing a ship are *Speed* and *Cost*, and *Speed* is determined to be 40 percent of the decision, *Cost* 60 percent, and each ship's value are as listed in Table 2.12.

Table 2.12: Constructed parameters for the three ships used in the rank reversal example decision: Which is the best ship?

	Cost (0.40)	Speed (0.60)
Ship I	\$20M	30
Ship II	\$30M	20
Ship III	\$10M	46

To simplify the math, comparisons of ships are done by dividing the *Cost* or *Speed* of each ship in the comparison by the highest value in the comparison. If Ship I and Ship II are compared without Ship III, the result would look like Table 2.13. Note that Ship I is considered superior to Ship II.

Table 2.13: Calculated global priorities for comparing Ship I and Ship II in the rank reversal example. *Cost* is normalized to \$30M, and *Speed* to 30.

	Cost (0.40)	Speed (0.60)	Global Priority
Ship I	0.67	1.00	**0.867**
Ship II	1.00	0.67	**0.800**

If all three ships are compared, the result would look like Table 2.14. Now Ship II is superior to Ship I. This is an example of rank reversal.

Table 2.14: Calculated global priorities for comparing all three ships in the rank reversal example. Here, *Cost* is normalized to $30M, and *Speed* to 46.

	Cost (0.40)	Speed (0.60)	Global Priority
Ship I	0.67	0.65	**0.657**
Ship II	1.00	0.43	**0.660**
Ship III	0.33	1.00	0.733

Many papers and articles have been written about this rank reversal phenomenon, on both sides of the discussion. Again, it is Mr. Holder who gives valuable insight to the problem:

> The problem of rank reversal arises because of the insistence that score vectors are normalized, either so that components sum to unity or so that the largest component is unity, before composition with weights, and because weights are elicited without reference to scales for performance against criteria. (Holder 1990, p. 1075)

When the attributes of the alternatives are scored, and those scores are normalized, they will, for instance, be on a zero to one scale. When a new option is introduced, that option, if it falls outside of the previous scale, will change the old scale. This can result in rank reversal, as seen in the above example, if the AHP is not used properly. The best way to fix this issue is to avoid changing the scale. If a new alternative is inserted into the process, it should be rated on the original scale, even when above or below the scale. In the rank reversal example this would mean using Table 2.15 as the corrected ranking.

Table 2.15: Corrected global priorities for comparing all three ships in the rank reversal example. Note, the *Speed* criteria is now normalized to 30, the same value as in Table 2.13'.

	Cost (0.40)	Speed (0.60)	Global Priority
Ship I	0.67	0.65	**0.867**
Ship II	1.00	0.43	**0.800**
Ship III	0.33	1.00	1.513

2.4 In Chapter III

In the next chapter, a sample decision that uses the AHP will be presented. In addition, the method by which this thesis will analyze the decision will be examined in detail.

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CHAPTER 3:
Methodology

To illustrate information that can be gained by analyzing the output of an AHP decision, this thesis looks at two real-world case studies. The first is a classified case study, covered in Appendix A. The second, discussed here, is based on a 1990 article *A Tradeoff Analysis For Rough Terrain Cargo Handlers Using The AHP: An Example Of Group Decision Making*, in which Dr. Jonathan Bard and Dr. Stephen Sousk compared three Rough Terrain Cargo Handlers (RTCH) for the U.S. Army. The evaluation team in this case study consisted of five managers and engineers from the Belvoir Research, Development and Engineering Center in Fort Belvoir, VA. The team worked with an objective hierarchy that contained 12 attributes (Bard & Sousk, 1990).

3.1 Case Study - Description

In the 1990s, the U. S. Army was making an effort to reduce risk and boost the productivity of material handling crews. At the time the article was written, the U. S. Army was using three different-sized rough terrain forklifts with maximum lifting capacities of 4,000 lbs., 6,000 lbs., and 10,000 lbs. each.

These vehicles were similar in design and performance to those used by industry, and, at best, could reach speeds of 20 mph. For the most part, this meant that the fleet was not self-deployable; i.e., it could not keep pace with a convoy on most surfaces. As a consequence, additional transportation resources were required for relocation between job sites. This restriction severely limited the unit's maneuverability and thus survivability on the battlefield. Additionally, cargo handling personnel had issues related to safety in hazardous nuclear, biological and chemical environments. Although protective gear was available, the operators' effectiveness was severely hampered by its use. Heat exhaustion, vision impairment, and the requirement for frequent gear changes were the most commonly cited problems (Bard & Sousk, 1990).

At the time, the U.S. Army was investigating the use of robotics to perform many of the dangerous and labor-intensive functions normally undertaken by enlisted personnel. To this end, a number of robotics programs were initiated: developing a Universal Self-Deployable Cargo Handler (USDCH) at Belvoir Research, Development, and Engineering Center; developing a

Field Material Handling Robot (FMHR) for the Human Engineering Laboratory, and prototyping an Advanced Robotic Manipulator System (ARMS) for the Defense Advanced Research Projects Agency (Bard & Sousk, 1990).

3.1.1 Case Study - Hierarchy

The team of experts assembled for this decision identified four dominant root factors, with 12 sub-factors that contributed to the overall outcome. The root factors and sub-factors are listed here:

1. Performance
   - Mission Objectives: How well does the alternative successfully complete the mission?
   - Reliability, Availability, and Maintainability (RAM): How well does the alternative meet military requirements and standards for RAM?
   - Safety: How well does the alternative protect the crew?
2. Risk
   - System Integration: How well does the alternative sync with other machinery?
   - Technical Performance: How does the alternative's technical specifications (speed, lift capability, etc.) compare to the requirements of the mission?
   - Cost Overrun: How likely is the alternative to have a cost overrun?
   - Schedule Overrun: How likely is the alternative to have a schedule overrun?
3. Cost
   - Research, Development, Test, and Evaluation (RDTE): How much is RDTE expected to cost for the alternative?
   - Life-Cycle Cost: What is the expected life cycle cost of the alternative?
4. Program Objectives
   - Implementation Timetable: How soon can the alternative be operational?
   - Technological Opportunities: What new technologies could be developed in conjunction with the alternative?
   - Acceptability: How well is the program accepted, by both user and maker?

To simplify the process, the experts were only asked to compare the 4 root factors for the process hierarchy. Table 3.1 re-creates the resulting pairwise comparison matrix as reported in the original article (Bard & Sousk, 1990). In order to validate the process that was used in the article, the data given for the pairwise comparisons were used to calculate the global priority weights, λ_{max}, Consistency Index, *CI*, and Consistency Ratio, *CR*. A minor discrepancy was found in the article's reported value of 0.097 for the *CI*, which corresponds to this author's calculated value of *CR*. This makes no difference to the integrity of the reported results in the article.

3.1.2 Case Study - Alternatives

According to Drs. Bard and Sousk, in using the AHP to choose amongst alternatives, the Army's process would result in a vehicle that would be:

- able to substitute for the existing 4,000 lb., 6,000 lb., and 10,000 lb. forklifts, while maintaining current material handling capabilities;
- capable of unaided movement (self-deployability) between job sites at convoy speeds in excess of 40 mph;
- capable of determining if cargo is contaminated by nuclear, biological, or chemical agents;
- capable of handling cargo in all climates, and under all contamination conditions;
- transportable by C-130 and C-141B aircraft;
- operable in the near term as an expandable semi-autonomous man-machine system;
- capable of robotic cargo engagement; and,
- remotely operable from up to one mile away (Bard & Sousk, 1990).

To this end, three alternatives were chosen (Bard & Sousk, 1990) for the analysis:

1. **Baseline** - The existing system, the 4,000, 6,000, and 10,000 lb rough terrain forklifts augmented with a 6,000 lb variable reach vehicle.
2. **Upgraded System** - The baseline models upgraded to be self-deployable.
3. **USDCH** - A teleoperable, robotic-assisted USDCH with micro-cooling for the protective gear, and the potential for full autonomy.

Table 3.1: Pairwise comparisons of the factor in the U. S. Army case study, with the resulting weights given in the far right column. Note, the *CI* reported in the original article (0.097) corresponds to what this thesis calls a *CR* (After Bard and Sousk 1991).

	Performance	Risk	Cost	Program Objective	**Global Priority**
Performance	1.00	5.00	3.00	4.00	0.517
Risk	0.20	1.00	0.17	0.33	0.059
Cost	0.33	6.00	1.00	4.00	0.306
Program Objective	0.25	3.00	0.25	1.00	0.118
$\lambda_{max} = 4.262$			$CI = 0.097$		

A graphical depiction of the decision process is given in Figure 3.1.2.

3.1.3 Case Study - Results

The three alternative cargo handlers were compared under each of the four factors (Performance, Risk, Cost, and Program Objective) to provide the comparison matrix reproduced in Table 3.2. Note that the values in each column total to one, indicating that these reported values are normalized. Included in Table 3.2 is also the Global Priority and Rank for each alternative, which is the final result of the AHP.

3.2 Thesis - Method

To garner useful information from this case study, the following was done:

- A comparison matrix was generated based on the article's calculated weights, to show the true comparisons used in the decision.
- A comparison matrix was generated based on the article's calculated results, to show how the alternatives compare to one another.
- Each alternative was removed from the list to test for rank reversal.
- A fourth alternative was added to the list of alternatives to test for rank reversal.
- A linear program was employed to show how a new alternative can be inserted anywhere into the list of alternatives with minimized resource allocation.

In addition, this thesis extrapolated the techniques above to illustrate how they can be used to the benefit of the side doing the analysis.

Table 3.2: Comparisons of each alternative and how it measures up to each factor. Included are the final tallied results for each alternative's global priority, and associated rank (After Bard and Sousk 1991).

Alternative	Performance	Risk	Cost	Program Objective	Global Priority	**Rank**
USDCH	0.691	0.067	0.299	0.705	0.536	1
Baseline	0.142	0.704	0.384	0.133	0.248	2
Upgrade	0.167	0.229	0.317	0.162	0.216	3

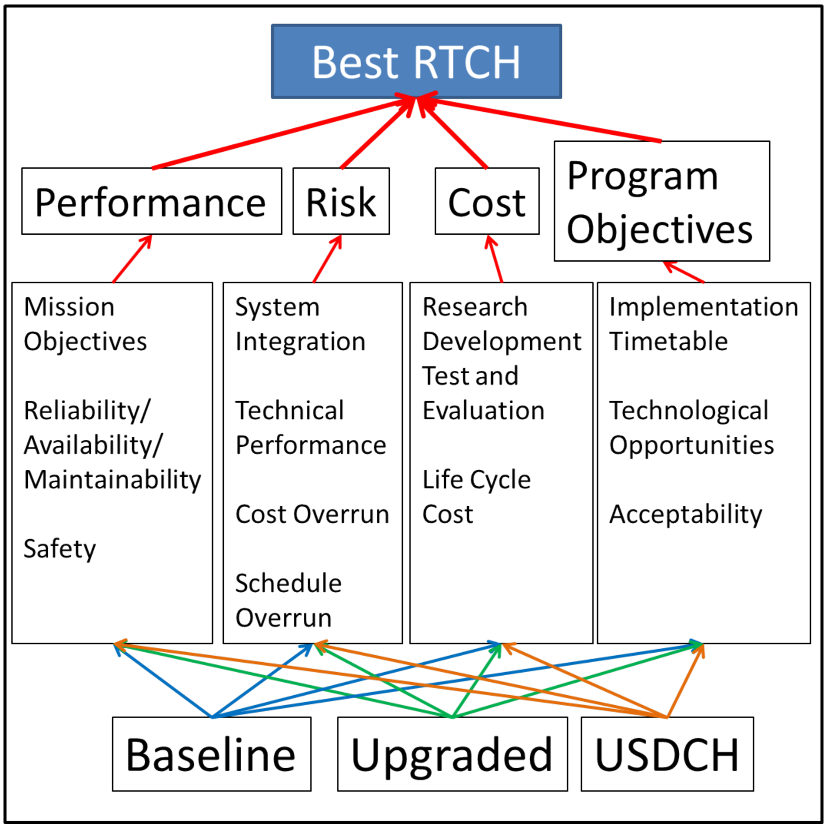

Figure 3.1: Representation of the AHP decision construct for the case study. Depicted are the 12 sub-factors that contribute to the four final factors used for the decision, as well as the three alternatives (After Bard and Sousk 1991).

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CHAPTER 4:
Analysis Results

For the results of the classified case study, refer to Appendix A. The analysis of the unclassified case study described in Chapter III netted two basic results. First, it allowed us to gain insights into the decision-maker's thoughts, either to verify preexisting assumptions, or to establish new ones. Second, the analysis allowed us to find ways to use the information gained to our advantage. This chapter details these two aspects. For the purpose of this case study, this author will assume the role of a manufacturer (Company A) who is competing with another manufacturer (Company B) to get the U.S. Army to buy its cargo handler model, and will use analysis of the decision presented in Chapter III as a means to further that end.

4.1 Analysis - What Can We Learn from the Decision Results?

The AHP lends itself well to gleaning information about the decision-maker who uses the process. First, the decision-maker's priorities are laid out in the reported factor weights and final results. In addition, mathematical manipulation of these numbers yields a true comparison matrix showing precisely how each of the factors or alternatives compare to each other, in the eyes of the decision-maker. Finally, the results can be tested for susceptibility to rank reversal; this is an important aspect of using the decision to one's advantage, as understanding of susceptibility to rank reversal can aid in manipulating future decisions given additional inputs.

4.1.1 Priorities

The basic concept of the AHP is to distill the priorities of a panel of subject matter experts into a single vector of weights. The weight vector from the U.S. Army case study appears in Table 4.1.

Table 4.1: Reported factor weights in the U.S. Army case study (After Bard and Sousk 1991).

FACTOR	WEIGHT
Performance	0.517
Cost	0.306
Program Objective	0.118
Risk	0.059

Some basic information can be learned just from inspecting this vector. First, the priority order of the factors is self evident. Here, *performance* is most important, and *risk* is least. As a manufacturer, this will lead Company A to spend more resources in an important area like *performance* in order to increase perceived overall quality in the eyes of this decision-maker. Additionally, the final output of the AHP, the result vector, is itself a list of priorities presented by the decision-maker. Table 4.2 shows the result vector from the U.S. Army case study.

These priority lists are very basic and quite easily attained, and it is good to keep in mind that these are an intrinsic part of the AHP that will be available from every decision made using that process.

4.1.2 True Comparisons

The next step in getting insight into a decision-maker's mind is to determine just how each factor or alternative compares to its peers. This is done by creating a matrix similar to Table 3.1, but instead of using subjective opinions to create the values, the value is calculated using the weight vector in Table 4.1 and the Equation 4.1.

$$ComparisonValue = \frac{ColumnFactorWeight}{RowFactorWeight} \tag{4.1}$$

The resulting matrix is given in Table 4.3. To show the result as it conforms to Dr. Saaty's nine point scale, the values have been rounded in Table 4.4.

From these tables it can be seen that, according to the experts chosen by the U.S. Army for this decision:

- Performance is between *equally important* and *slightly more important* than Cost
- Performance is between *slightly more important* and *more important* than Program Objective

Table 4.2: Reported final scores for the three alternatives in the U.S. Army case study (After Bard and Sousk 1991).

ALTERNATIVE	SCORE
USDCH	0.536
Baseline	0.248
Upgrade	0.216

Table 4.3: Calculated table of actual pairwise comparisons based on reported weights (After Bard and Sousk 1991).

	Performance	**Cost**	**Program Objective**	**Risk**
Performance	1.00	1.69	4.38	8.76
Cost	0.59	1.00	2.59	5.19
Program Objective	0.23	0.39	1.00	2.00
Risk	0.11	0.19	0.50	1.00

Table 4.4: Table of rounded pairwise comparisons based on reported weights, showing relations to Dr. Saaty's nine point scale (After Bard and Sousk 1991).

	Performance	**Cost**	**Program Objective**	**Risk**
Performance	1	2	4	9
Cost	$\frac{1}{2}$	1	3	5
Program Objective	$\frac{1}{4}$	$\frac{1}{3}$	1	2
Risk	$\frac{1}{9}$	$\frac{1}{5}$	$\frac{1}{2}$	1

- Performance is *absolutely more important* than Risk
- Cost is *slightly more important* than Program Objective
- Cost is *more important* than Risk
- Program Objective is between *equally important* and *slightly more important* than Risk

This is valuable information that is used to build a picture of how this decision-maker thinks. For instance it can be asserted from this information, that the U.S. Army's panel of experts considers the likelihood of a manufacturer's cost overrun (*risk*) to be negligibly important when compared to the a cargo handler's operational performance (*performance*). As a manufacturer, Company A may decide to forgo careful consideration of the cost of the cargo handler manufacturing process because the U.S. Army considers that to be of low importance.

This same process of turning the vector into a list of comparisons can be done with the alternatives in the decision as well. The pairwise comparisons for the alternatives appear in Table 4.5, with the rounded form in Table 4.6.

Table 4.5: Pairwise comparisons of the three alternatives in the sample decision (After Bard and Sousk 1991).

	USDCH	**Baseline**	**Upgrade**
USDCH	1.00	2.16	2.48
Baseline	0.46	1.00	1.15
Upgrade	0.40	0.87	1.00

Table 4.6: Rounded pairwise comparisons of the three alternatives in the sample decision (After Bard and Sousk 1991).

	USDCH	**Baseline**	**Upgrade**
USDCH	1	2	2
Baseline	$\frac{1}{2}$	1	1
Upgrade	$\frac{1}{2}$	1	1

Again, from these numbers it is seen that:

- The USDCH is between *equally acceptable* and *slightly more acceptable* than the Baseline model.
- The USDCH is between *equally acceptable* and *slightly more acceptable* than the Upgraded model.
- The Baseline model is *equally acceptable* as the Upgraded model.

As a competing manufacturer, Company A now has an idea of how well its cargo handler will have to perform to make it competitive. In addition, Company A can begin to build a picture of how hard it will be to make one that is *more acceptable* than the USDCH, the current favorite.

4.1.3 Rank Reversal Testing

The potential for rank reversal in a decision is important to know. If the potential is evident, a process can be developed to exploit this weakness. If the potential is not present, the introduction of new alternatives to the decision will produce predictable results. Knowledge of the potential for rank reversal, therefore, is a valuable tool if one is trying to alter the decision to one's advantage. It allows one side to have some control over another side's decision-making process. Recall from Chapter II that there are two general ways to induce rank reversal: removing an alternative, and adding an alternative. These methods will both be explored for the U.S. Army case study decision.

Removing an Alternative

The procedure for removing an alternative from the list is simple in a decision with three alternatives. One alternative is removed, and the two remaining are subject to the same procedure outlined in Chapter II for producing a result. This process is detailed for the removal of the USDCH alternative in Table 4.7. Note, these tables depict two subtables, the top shows what alternative was added or removed, and the bottom shows how the new alternative list looks when normalized. For decisions with more than three variables, sets of alternatives would need to be removed sequentially for full rigorous analysis.

Table 4.7: Testing for rank reversal by removing the USDCH alternative from the list of alternatives (After Bard and Sousk 1991).

Alternative	Performance	Risk	Cost	Prgm Obj	Global Priority	Rank
~~USDCH~~	~~0.69~~	~~0.07~~	~~0.30~~	~~0.71~~	~~0.536~~	~~1~~
Baseline	0.14	0.70	0.38	0.13	0.248	2
Upgrade	0.17	0.23	0.32	0.16	0.216	3
Renormalized List	Performance	Risk	Cost	Prgm Obj	Global Priority	Rank
Baseline	0.46	0.75	0.55	0.45	0.503	1
Upgrade	0.54	0.25	0.45	0.55	0.497	2

This same process was used for the other two alternatives as well, with the result that no rank reversal occurs when any of the alternatives are removed.

Adding an Alternative

The next step in testing for rank reversal is to add an alternative in order to induce reversal. As a manufacturer trying to have an effect on this decision, it would be valuable to be able to change the outcome of the decision, not just in where Company A's product will fall, but in the order of the other products as well. The common additions to the alternative list that cause rank reversal include adding a copy of each alternative, adding an alternative that consists of the highest or lowest attributes, and an alternative that has attributes of 1.5 on the zero to one scale used for the current alternatives. An example of this for adding a copy of the USDCH alternative is found in Table 4.8, the remaining options can be found in Appendix 2. In order to test for rank reversal with an added alternative, the new list of alternatives will need to be renormalized. Table 4.8 shows both the list of alternatives with the raw new alternative, and the list of alternatives with renormalized values.

Inspection of these tables show that there is no evidence of rank reversal. This decision is, therefore, very resistant to rank reversal. As a manufacturer, Company A can use that knowledge to its advantage by using the predictability of the decision. A method of doing this is outlined in the next section.

4.2 Using the Information Gained

The next step in analyzing this decision, having extracted the information about both the decision-maker's priorities and the decision's resistance to rank reversal, is to use the decision to Company A's advantage as a manufacturer to get the U.S. Army to buy their cargo handler before

Table 4.8: Testing for rank reversal by copying the USDCH alternative from the list of alternatives (After Bard and Sousk 1991).

Alternative	Performance	Risk	Cost	Prgm Obj	Global Priority	Rank
USDCH	0.69	0.07	0.30	0.71	0.536	1
Baseline	0.14	0.70	0.38	0.13	0.248	3
Upgrade	0.17	0.23	0.32	0.16	0.216	4
Copy 1	0.69	0.07	0.30	0.71	0.536	1
Renormalized List	Performance	Risk	Cost	Prgm Obj	Global Priority	Rank
USDCH	0.41	0.06	0.23	0.41	0.334	1
Baseline	0.08	0.66	0.30	0.08	0.182	3
Upgrade	0.10	0.21	0.24	0.10	0.150	4
Copy 1	0.41	0.06	0.23	0.41	0.334	1

buying Company B's. The premise of this step is to use the information readily available in the AHP report, factor weights and alternative values, and Company A's knowledge of the cost of making an alternative product, to develop an alternative that accomplishes the goal. For the sake of this case study, assume that the goal is to develop a cargo handler that the U.S. Army will consider to be superior to the others listed in their report. The process in doing this will be to first assign costs to each factor; second, to determine the relative value of Company A's cargo handler; and third, to solve for the factor contributions Company A will have to make to develop this cargo handler.

The first step is done by equating a utility of some kind to devoting resources to each factor in the decision. For some factors like *Cost*, this will be an easy, one-to-one utility function. For others like *Risk*, the function will be more complicated. For simplicity, the cost of devoting any resources to a factor will initially be defined as equal across the factors in this decision.

The second step is a direct decision. Does Company A want its cargo handler to be seen as equal to the best? This would force Company B to invest more resources to win, making their product more expensive. Does Company A want their cargo handler to seem much better than the current best, to win the contract outright? Company A chooses its cargo handler's place in the list, and determines what value that should have according to the published results. For example purposes, Company A wants its cargo handler to be ten percent superior to the best alternative on the list, the USDCH.

The final step in this process is to construct a linear program to solve for the value of the contribution Company A will have in each factor in order to minimize the cost of building an alternative. This is done easily in any mathematical program that solves linear equations, including Excel, which was used for this thesis. The variable to be minimized is the sum of the contributions, or the resource allocation for the process. The given values are the published factor weights and alternative contributions. In addition, the costs for each factor are preset, and have been determined here to be equal; thus they are set to one. The values to be adjusted are the contribution values, and they are constrained to be less than one, by AHP convention. The final constraint is that the output of the new alternative that is added must be ten percent superior to the USDCH's output. Similar to the procedure for testing for rank reversal, the new list of alternatives' contributions will need to be renormalized. Table 4.9 shows the result of the linear programmed minimization of resource allocation described above.

Table 4.9: Result of minimizing resource allocation to a new alternative to achieve a specified rank. Here the 'cost' of each factor is equal, and the target rank is 1 (After Bard and Sousk 1991).

Factor (Weight)	Performance (0.517)	Risk (0.059)	Cost (0.306)	Prgm Obj (0.118)		
Resource Cost	1.00	1.00	1.00	1.00	**Resource Allocation**	
Contribution	1.00	0.00	0.45	0.04	**1.49**	
Alternative	Performance	Risk	Cost	Prgm Obj	Global Priority	Rank
USDCH	0.69	0.07	0.30	0.71	0.536	2
Baseline	0.14	0.70	0.38	0.13	0.248	3
Upgrade	0.17	0.23	0.32	0.16	0.216	4
Co.A Alternative	1.00	0.00	0.45	0.04	0.660	1
Alternative	Performance	Risk	Cost	Prgm Obj	Global Priority	Rank
USDCH	0.35	0.07	0.21	0.67	0.326	2
Baseline	0.07	0.70	0.26	0.13	0.174	3
Upgrade	0.08	0.23	0.22	0.16	0.142	4
Co.A Alternative	0.50	0.00	0.31	0.04	0.358	1

Color Sceme	Set Value	Calculated Value	Target Value	Minimized Value

This result gives Company A an idea of how expensive it will be to manufacture an alternative that is 10 percent superior to the best one in the decision. If this solution is feasible and acceptable Company A can commence making the cargo handler. This will also give Company A direction in how to design their cargo handler. Table 4.9 shows that focusing on increased *performance* (1.0) and reduced *cost* (0.45) will net better gains than *risk* (0.00) or *program objective* (0.04). This may translate into Company A's CEO ordering the COO to design a better performing cargo handler at a cheaper cost, but ignoring overruns, and its ability to fulfill the mission. In addition, this linear program allows a manufacturer to easily change the cost of each factor to see how the overall resource allocation will change. For instance, Table 4.10 shows how the allocation of resources change if the resource cost of factor *Performance* is five time higher than the other three. Note that it is now more beneficial to concentrate on *cost* (1.00) and *program objective* (0.62) than *performance* (0.51) and *risk* (0.00). Here, Company A's CEO can decide to reduce the cost to the consumer, as it is the most beneficial factor to manage.

Table 4.10: Result of minimizing resource allocation to a new alternative to achieve a specified rank. Here the 'cost' of each factor is offset to show how the results can change, and the target rank is still 1 (After Bard and Sousk 1991).

Factor (Weight)	Performance (0.517)	Risk (0.059)	Cost (0.306)	Prgm Obj (0.118)		
Cost	5.00	1.00	1.00	1.00	**Resource Allocation**	
Contribution	0.51	0.00	1.00	0.62	**4.153**	
Alternative	Performance	Risk	Cost	Prgm Obj	Global Priority	Rank
USDCH	0.69	0.07	0.30	0.71	0.536	2
Baseline	0.14	0.70	0.38	0.13	0.248	3
Upgrade	0.17	0.23	0.32	0.16	0.216	4
Co.A Alternative	0.51	0.00	1.00	0.62	0.597	1
Alternative	Performance	Risk	Cost	Prgm Obj	Global Priority	Rank
USDCH	0.46	0.07	0.15	0.44	0.338	2
Baseline	0.09	0.70	0.19	0.08	0.159	3
Upgrade	0.11	0.23	0.16	0.10	0.131	4
Co.A Alternative	0.34	0.00	0.50	0.38	0.372	1

Color Scheme	Set Value	Calculated Value	Target Value	Minimized Value

It can be seen from Table 4.10 that resource allocation has risen, but not linearly, with a rise in *Performance* cost. Again, this will lead a manufacturer to make critical decisions about a production process.

4.2.1 Taking Advantage of the Information Gained

The previous section discussed ways to use the output of an AHP decision, in an effort to get a new alternative accepted. This section will discuss ways that this procedure can be expanded to further the advantageous use of the output of an AHP decision. It may be advantageous in different scenarios to alter the way that a rival decision-maker perceives a competitive product. This alteration can be accomplished with a combination of knowing of a decision that has been made using the AHP and misinforming the decision-maker of the vital statistics of one's product. There are two ways to misinform a decision-maker: get the decision-maker to underestimate the product, or get them to over-estimate it. These two techniques can be used for both short-term and long-term strategies. For demonstration purposes, the author will still assume the role of a manufacturer (Company A) who is competing with another manufacturer (Company B) to secure a contract selling cargo handlers to the DOD, using the decision made by the U.S. Army as a reference.

Underestimation - Short-Term

Underestimation may be useful when Company A knows that Company B will actively pursue research to match any product that Company A markets. In order to stay one step ahead, Company A may want to force Company B to underestimate the capabilities of Company A's cargo handler. Company B will mistakenly believe its system to be competitive, and will dedicate inadequate resources to design, research, and develop an inferior system that will not be selected for procurement. A simple application of this would be to use a linear programming technique similar to the one used in the previous section. The desired global priority of the product is chosen, and a linear program is run to determine what the contributions for each factor would be. Additionally, as this new process is to misinform, more constraints can be placed on the linear program, for instance requiring that the new alternative's *Performance* be no higher than the current highest. Table 4.11 shows the result of using this process, with this added constraint. Now Company A's CEO may order public affairs personnel to show that Company A's cargo handler does not outperform Company B's, resulting in the desired underestimation.

The result in Table 4.11 means that if Company A wants to get Company B to underestimate their product, they need only to classify the cargo handler's statistics that are above the calculated values.

Overestimation - Short-Term

In addition to underestimation, overestimation can be a valuable tool as well. Consider the situation where Company A wants Company B to believe that Company A's product performs better than it does. This may be to encourage Company B to withdraw from a contract competition, mistakenly thinking their product cannot compete, or to needlessly spend more resources further developing their own product. The result is that Company B, if they remain in the competition, will present a product that is either too costly to be chosen, or too costly for them to profit from when chosen. Either way, Company A will want to deceive Company B into believing their cargo handler performs better than it actually does. The end result would be inflated values for the statistics of Company A's cargo handler. Exactly how inflated these numbers will need to be can be determined using the same linear programming method used in the under-

Table 4.11: Result of a Linear Program designed to misinform a rival using underestimation, claiming that the new alternative is ten percent better in global priority to the current highest global priority (After Bard and Sousk 1991).

Factor (Weight)	Performance (0.517)	Risk (0.059)	Cost (0.306)	Prgm Obj (0.118)		
Cost	1.00	1.00	1.00	1.00	**Resource Allocation**	
Contribution	0.69	0.00	0.62	0.15	**1.463**	
Alternative	Performance	Risk	Cost	Prgm Obj	Global Priority	Rank
USDCH	0.69	0.07	0.30	0.71	0.536	2
Baseline	0.14	0.70	0.38	0.13	0.248	3
Upgrade	0.17	0.23	0.32	0.16	0.216	4
Co.A Alternative	0.69	0.00	0.62	0.15	0.565	1
Alternative	Performance	Risk	Cost	Prgm Obj	Global Priority	Rank
USDCH	0.41	0.07	0.18	0.62	0.344	1
Baseline	0.08	0.70	0.24	0.11	0.171	3
Upgrade	0.10	0.23	0.20	0.14	0.141	4
Co.A Alternative	0.41	0.00	0.38	0.13	0.344	1

Color Scheme	Set Value	Calculated Value	Target Value	Minimized Value

estimation section. This use, however, would see the new alternative's global priority set to something larger than the original rank one's global priority. As an example, Table 4.12 shows how this would look, setting the new alternative's global priority equal to two times the original best option's global priority.

Inspection of the result in Table 4.12 shows that the new alternative's value for *Performance* will need to be shown to be 1.78 on the previous scale used for the original three alternatives. This gives an indication as to how well Company A's new alternative will need to be shown to perform to achieve the desired result.

4.2.2 Underestimation and Overestimation - Long-Term Uses

The long-term strategic uses of underestimation and overestimation are based on the desire to make a rival decision-maker either neglect to spend resources on needed research and development, or to needlessly spend resources on unnecessary research and development.

Beginning with underestimation, Company A, who through their analysis of the U.S. Army's cargo handling decision, is confident that they understand the required capabilities for cargo handlers, can publicly downplay the most important capabilities actual importance. The intent of this would be to lull Company B into not focusing research and development on these key

Table 4.12: Result of a Linear Program designed to misinform a rival using overestimation, claiming the new alternative is twice as good as the old rank 1 (After Bard and Sousk 1991).

Factor (Weight)	Performance (0.517)	Risk (0.059)	Cost (0.306)	Prgm Obj (0.118)		
Cost	1.00	1.00	1.00	1.00	**Resource Allocation**	
Contribution	1.78	0.00	0.76	0.34	**2.880**	
Alternative	Performance	Risk	Cost	Prgm Obj	Global Priority	Rank
USDCH	0.69	0.07	0.30	0.71	0.536	2
Baseline	0.14	0.70	0.38	0.13	0.248	3
Upgrade	0.17	0.23	0.32	0.16	0.216	4
New Alternative	1.78	0.00	0.76	0.34	1.194	1
Alternative	Performance	Risk	Cost	Prgm Obj	Global Priority	Rank
USDCH	0.25	0.07	0.17	0.53	0.246	2
Baseline	0.05	0.70	0.22	0.10	0.147	3
Upgrade	0.06	0.23	0.18	0.12	0.114	4
New Alternative	0.64	0.00	0.43	0.25	0.493	1

Color Scheme	Set Value	Calculated Value	Target Value	Minimized Value

capabilities, thus giving Company A a competitive advantage.

Overestimation, a more aggressive strategy, would focus more on convincing Company B of the importance of a needless future capability, one not required at all to be competitive. Company B would focus their research and development on an unnecessary capability, allowing Company A to quietly pursue research and development of the key capabilities desired by the U.S. Army.

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CHAPTER 5:
Conclusion

5.1 Conclusion

Having the output of the Analytic Hierarchy Process of a decision-maker has many advantages. One is the ability to surmise precisely what the decision-maker's priorities are, both in the factors that make up the decision, and in the alternatives that have been considered in the actual decision as well. Further, knowing the mechanics of the AHP one is able to test the decision output for its susceptibility to rank reversal, the knowledge of which is useful. Finally, one can use the AHP decision output to gain an understanding of the decision-maker, allowing the decision-maker to be deceived using the techniques of underestimation, overestimation, or misinformation.

5.2 Future Work

Follow on work to this thesis would be testing the potential for garnering better information about a specific decision-maker by analyzing multiple AHP decision outputs from the same source.

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Appendix: Special Case Study

Refer to: Analysis of Decisions Made Using the Analytic Hierarchy Process - Special Case Study (CLASSIFIED DOCUMENT)

This special case study examines the use of the techniques described in this thesis as they apply to a decision made by a foreign military, and gives recommendations for implementation. For access to this addendum, please contact the Dudley Knox Library.

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